Transport coefficients and nPI methods

tc measure efficiency with which a conserved quantity is transported over ‘long’ distances
(long compared to microscopic relaxation scales)

effective kinetic theory:
small deviations from thermal equilibrium
weak coupling
$\longrightarrow$ using equilibrium FT tools

will show:
3PI effective theory $\longrightarrow$ same result (σ_{qed})

motivation:
[1] in principle can use nPI far from equib
[2] possibility to go beyond leading order (?)

nPI methods:

motivation:
sometimes standard pt is poorly convergent
try to improve convergence with non-pert techniques

(ex: gauge theories at high T
$\longrightarrow$ HTL effective theory)

far-from-equib dynamics
nPI Γ : self-consistent in terms of dressed n-pt fcns

2PI QED:

$$Z[J,\eta,\bar\eta,C,B]$$
$$= \int D[\mathcal{A}\Psi\bar\Psi]\text{Exp}\,\Big[i(S_{cl} + J_1\mathcal{A}_1 + \bar\eta_1\Psi_1 + \bar\Psi_1\eta_1$$
$$+ \frac{1}{2}C_{12}\mathcal{A}_1\mathcal{A}_2 + B_{12}\Psi_1\bar\Psi_2)\Big]$$

legendre transform

$$\begin{aligned}
\Gamma[\psi,\bar{\psi},A,S,D] &= S_{cl}[\psi,\bar{\psi},A] \\
&+\frac{i}{2}\mathrm{Tr}\,\mathrm{Ln}D_{12}^{-1}+\frac{i}{2}\mathrm{Tr}\left[(D^0_{12})^{-1}\left(D_{21}-D^0_{21}\right)\right] \\
&-i\mathrm{Tr}\,\mathrm{Ln}S_{12}^{-1}-i\mathrm{Tr}\left[(S^0_{12})^{-1}(S_{21}-S^0_{21})\right] \\
&+\Phi[S,D]
\end{aligned}$$

$S_{cl}[\psi,\bar{\psi},A]$ is the classical action

S_0 and D_0 are the free propagators

$\Phi[S,D]$ is the sum of all 2PI diagrams

EoM obtained from the stationarity of the action:

$$\frac{\delta\Gamma[\psi,\bar{\psi},A,S,D]}{\delta A}=0\,;\quad \frac{\delta\Gamma[\psi,\bar{\psi},A,S,D]}{\delta D}=0$$

$$\frac{\delta\Gamma[\psi,\bar{\psi},A,S,D]}{\delta \psi}=0\,;\quad \frac{\delta\Gamma[\psi,\bar{\psi},A,S,D]}{\delta \bar{\psi}}=0$$

$$\frac{\delta\Gamma[\psi,\bar{\psi},A,S,D]}{\delta S}=0$$

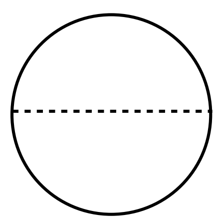
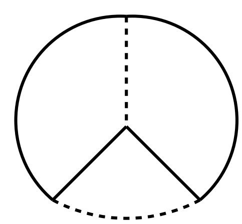
systematic non-pert approx by:
expand Φ (ex: loop or 1/N expansion)
solve EoM w/o further approx

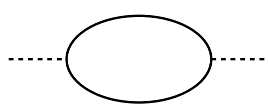
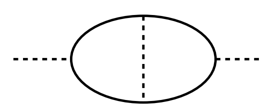
consv laws corresponding to global symmetries are respected (at any approx order) example:

$$\Gamma[\psi, \bar{\psi}, A, S, D] = S_{cl}[\psi, \bar{\psi}, A] + \Phi[S, D]$$
$$+\frac{i}{2}\mathrm{Tr}\,\mathrm{Ln} D_{12}^{-1} + \frac{i}{2}\mathrm{Tr}\left[(D_{12}^0)^{-1}\left(D_{21} - D_{21}^0\right)\right]$$
$$-i\mathrm{Tr}\,\mathrm{Ln} S_{12}^{-1} - i\mathrm{Tr}\left[(S_{12}^0)^{-1}(S_{21} - S_{21}^0)\right]$$

EoM has form of a dyson equation:

$$\frac{\delta\Gamma}{\delta D} = -D^{-1} + [\ (D^0)^{-1} - \underbrace{2i\frac{\delta\Phi}{\delta D}}_{\Pi}\ ] = 0$$

$\Phi(S,D) =$ i / 2  +i / 4 

$\Pi =$ i  + i 

PROBLEM:
truncations $\Rightarrow$ gauge dependence

wi depend on cancellations btwn different topologies (vertex corrections and self energy corrections)

2PI effective theory
$\longrightarrow$ corrected propagators but not corrected vertices
$\longrightarrow$ expect the ward identities are not satisfied

Strategy

introduce the resummed effective action
define w.r.t self-consistent solns of the propagators

$$\frac{\delta\Gamma[\psi,\bar{\psi},A,S,D]}{\delta S}\Big|_{\{S=\tilde{S}[\psi,\bar{\psi},A],\ D=\tilde{D}[\psi,\bar{\psi},A]\}}=0$$
$$\frac{\delta\Gamma[\psi,\bar{\psi},A,S,D]}{\delta D}\Big|_{\{S=\tilde{S}[\psi,\bar{\psi},A],\ D=\tilde{D}[\psi,\bar{\psi},A]\}}=0$$

substituting the self consistent solutions we obtain the resummed action:

$$\tilde{\Gamma}[\psi,\bar{\psi},A]=\Gamma[\psi,\bar{\psi},A,\tilde{S}[\psi,\bar{\psi},A],\tilde{D}[\psi,\bar{\psi},A]]$$

MOTIVATION FOR RESUMMED ACTION

[A] fcnal derivs of resummed action $\tilde{\Gamma}[\psi, \bar{\psi}, A]$
$\longrightarrow$ n-point functions for the 'external' fields
satisfy standard ward identities

[B] renormalizable
finite # of local medium independent counter-trms
J. Berges, S. Borsanyi, U. Reinosa, J. Serreau, Annals Phys. **320**, *344 (2005); U. Reinosa, J. Serreau, JHEP* **0607**, *028 (2006).*

Different possible defns of n-point functions:

$$V^{(n)}_{1,2,\cdots n} \sim \frac{\delta^n\,\tilde{\Gamma}(A)}{\delta A_1 \delta A_2 \cdots \delta A_n}\Big|_{A=\tilde{A}}$$

$$V^{(n)}_{1,2,\cdots n}$$

$$\sim \frac{\delta}{\delta A_1 \delta A_2 \cdots \delta A_{n-2}} \left(\frac{\delta \Phi(A,D)}{\delta D_{n-1,n}}\Big|_{D=\tilde{D}(A)} \right) \Big|_{A=\tilde{A}}$$

- other mixed definitions possible
- all definitions equivalent at exact level
- there are relations between the definitions
  (chain rule)

general idea:
integral eqns for the n-point fcns of external fields
kernels of integral eqns from fcnal derivs of Φ

define 'external' propagators:

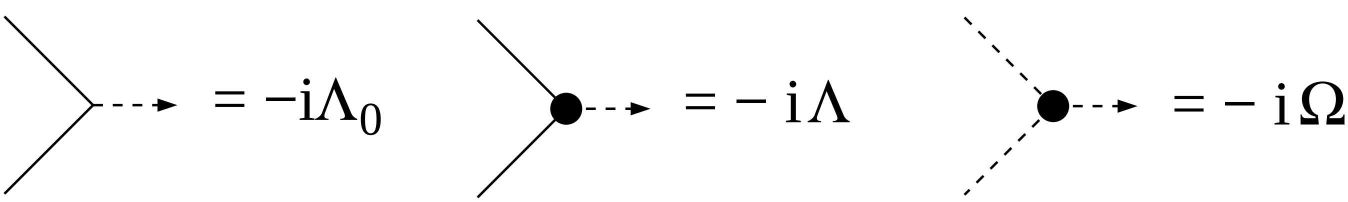
$$(D_{12}^{\text{ext}})^{-1} = \frac{\delta^2}{\delta A_2 \delta A_1}\tilde{\Gamma}\,; \quad (S_{12}^{\text{ext}})^{-1} = \frac{\delta^2}{\delta \psi_2 \delta \bar{\psi}_1}\tilde{\Gamma}$$

define vertices

$$\Lambda_{132}^{0} = -\frac{\delta^3\ S_{cl}}{\delta A_3 \delta \psi_2 \delta \bar{\psi}_1} = -\frac{\delta (S_{12}^{0})^{-1}}{\delta A_3}$$

$$\Lambda_{132} = -i\frac{\delta^2 \Phi}{\delta A_3 \delta S_{21}} = -\frac{\delta \tilde{S}_{12}^{-1}}{\delta A_3}$$

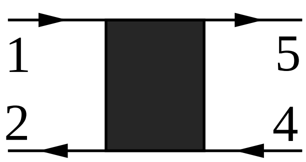
$$\Omega_{132} = i\frac{\delta \Phi}{\delta A_3 \delta D_{21}} = -\frac{1}{2}\frac{\delta \tilde{D}_{12}^{-1}}{\delta A_3}$$



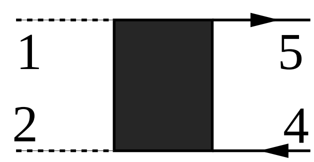
4-point functions:

$$M^{SS}_{54;21} = -\frac{\delta^2\Phi[\tilde{S},\tilde{D}]}{\delta\tilde{S}_{12}\delta\tilde{S}_{45}}\,; \quad M^{SD}_{54;21} = -2\frac{\delta^2\Phi[\tilde{S},\tilde{D}]}{\delta\tilde{D}_{12}\delta\tilde{S}_{45}}$$

$$M^{DS}_{54;21} = -2\frac{\delta^2\Phi[\tilde{S},\tilde{D}]}{\delta\tilde{S}_{12}\delta\tilde{D}_{45}}\,; \quad M^{DD}_{54;21} = 4\frac{\delta^2\Phi[\tilde{S},\tilde{D}]}{\delta\tilde{D}_{12}\delta\tilde{D}_{45}}$$



$= -i\,M^{SS}_{54;21}$



$= -i\,M^{SD}_{54;21}$

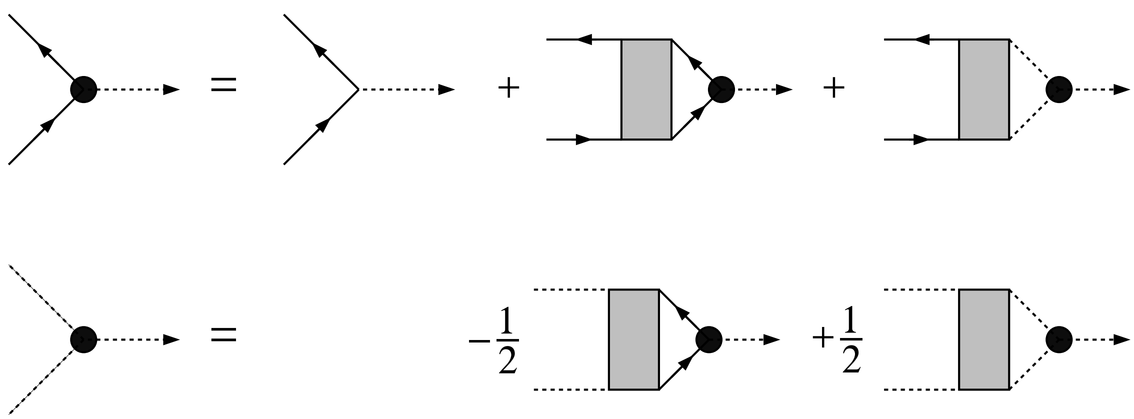
Bethe-Salpeter eqns

Bethe-Salpeter type equations for the vertices:
from fcnal derivatives of the dyson equations wrt A

$$\tilde{S}^{-1} = (S^0)^{-1} + \underbrace{i\frac{\delta\Phi}{\delta S}(S,D)|_{\tilde{S},\tilde{D}}}_{-\Sigma}$$

$$\tilde{D}^{-1} = \ (D^0)^{-1} - \underbrace{2i\frac{\delta\Phi}{\delta D})|_{\tilde{S},\tilde{D}}}_{\Pi}$$

graphically:



External Propagator:

$$(D^{\rm ext}_{12})^{-1} = \frac{\delta^2}{\delta A_2 \delta A_1} \tilde{\Gamma}[\psi, \bar{\psi}, A]$$

using the BS eqn $\longrightarrow$

$$(D^{\rm ext}_{12})^{-1} = (D^0_{12})^{-1} + i(\Lambda^0_{314} \tilde{S}_{44'} \Lambda_{4'23'} \tilde{S}_{3'3})$$

external propagator satisfies the usual wi:

$$\partial_1 (D^{\rm ext}_{12})^{-1} = 0$$

basic mechanism is simple:

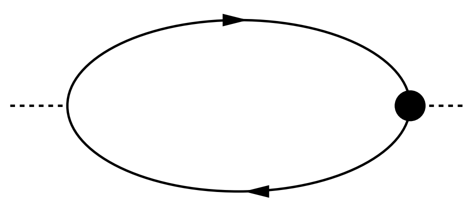
dyson equations contain s-channel resummations

BS eqns introduce t- and u-channels

$\longrightarrow$ crossing symmetry is restored

we extract the vertex part of the 2-point function:

$$\Pi^{\rm ext}_{12} = -i {\rm Tr}\, [\Lambda^0_1 \tilde{S} \Lambda_2 \tilde{S}]$$

$\Pi^{\rm ext}_{12} = \;\; -i$ 

Conductivity:
kubo formula:

$$\sigma = -\frac{1}{6e^2}\left(\frac{\partial}{\partial q_0} 2\,\mathrm{Im}\,\Pi^{ii}_{ret}(q_0,0)\right)|_{q_0\to 0}$$

∞ # terms contribute at the same order
(from low frequency limit in the kubo formula)

pairs of ret/adv propagators with same momenta
integrating a term $\int dp_0\, G^{ret}(P)G^{adv}(P)$
$\rightarrow$ a divergence called a 'pinch singularity'

regulate using resummed propagators
(finite width of thermal excitations)

$\rightarrow$ extra factors of the coupling in the denominators
$\rightarrow$ infinite set of graphs which contain products of pinching pairs that all need to be resummed

RESUMMATION PINCH SINGULARITIES

we use:

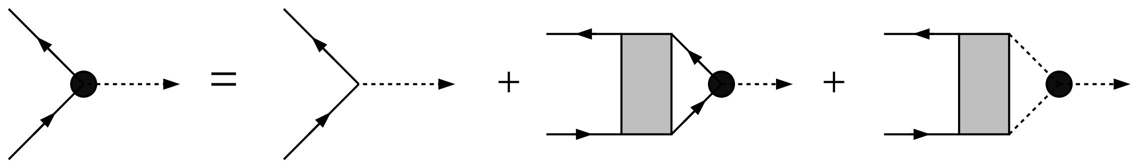
$$\Pi = -i\Lambda^0 S \Lambda S$$

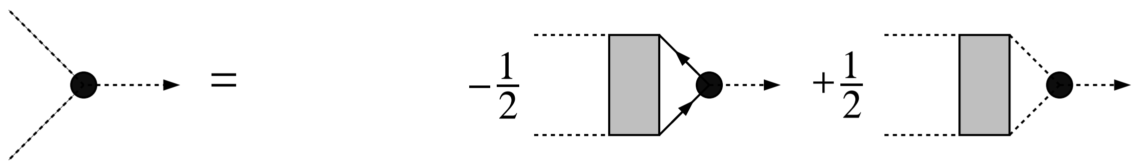
there is an int eqn for Λ that resums pinch terms

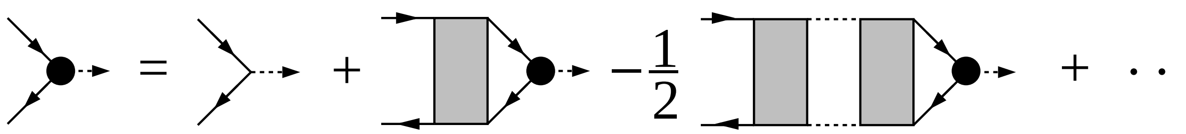
kernel is square of the matrix elements that correspond to the $2 \rightarrow 2$ scattering and production processes (AMY)

show this int eqn produced by the 2PI formalism

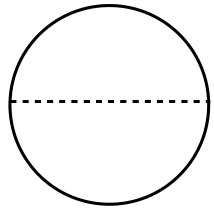
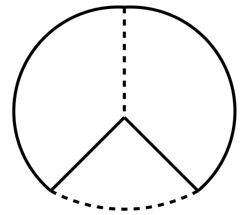
iterate BS eqns:



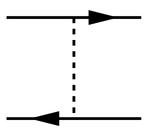
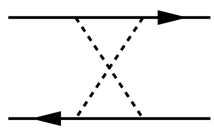
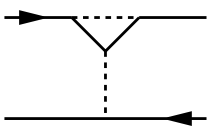
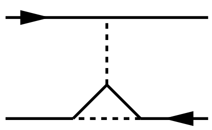


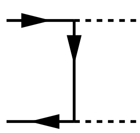
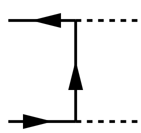


calculate M's from derivatives of Φ

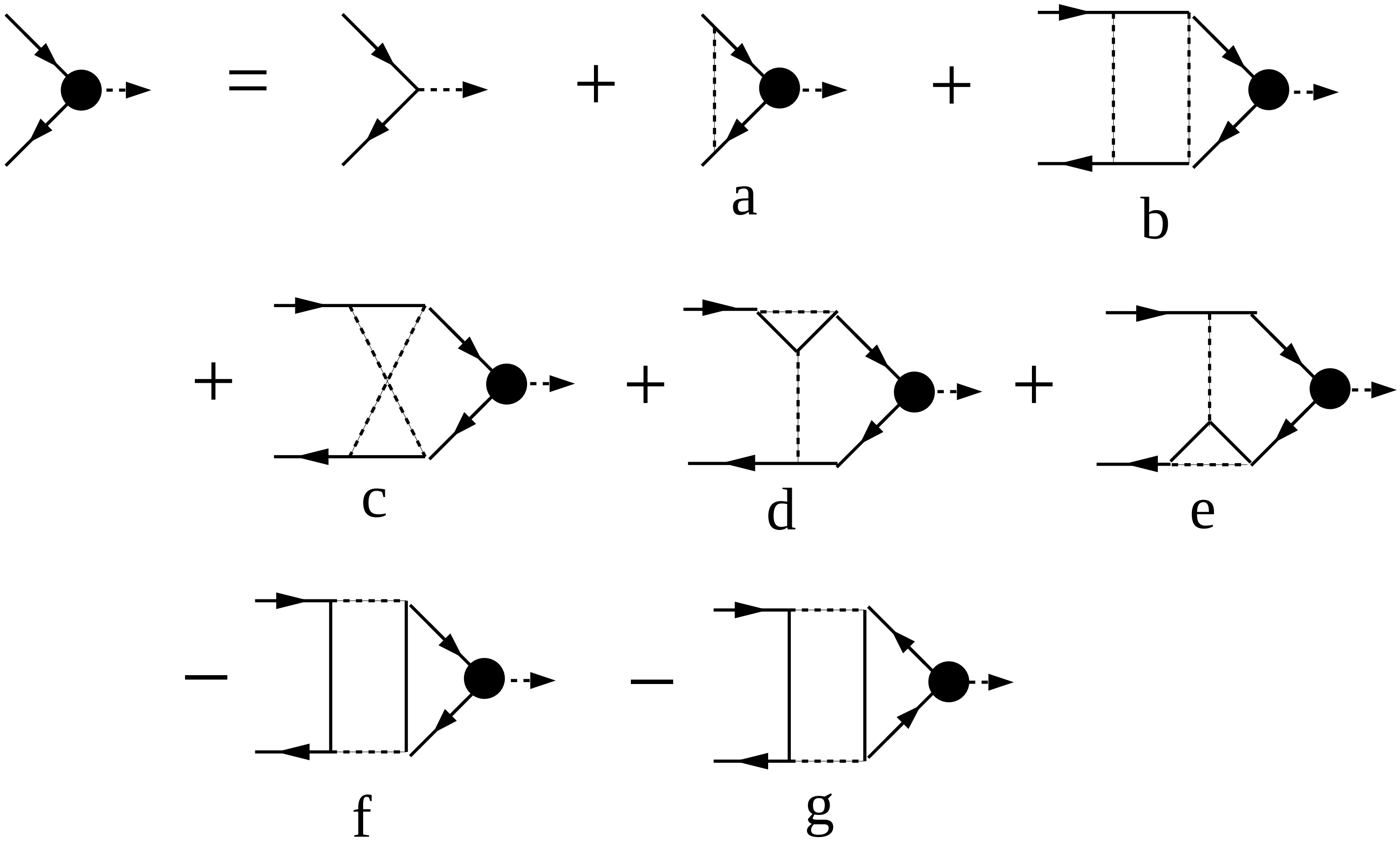
$\Phi(S,D) = \; i/2$  $+ i/4$ 

results:

$-iM^{SS} =$  $+$  $+$  $+$ 

$-iM^{SD} =$  $+$ 

integral eqn for Λ



integral eqn:

$$\mathrm{Re}\hat{\Lambda}^i(3,P) = \mathrm{Re}\,\hat{\Lambda}^i_0(3,P) \quad + \sum_{j\in\{a,b,c,d,e,f,g\}}$$

$$\frac{1}{2}\int dK\ \mathrm{Re}\,[\,\hat{M}^{(j)}(P,K)\,]\frac{\rho(K)}{2\mathrm{Im}\,\hat{\Sigma}(K)}\mathrm{Re}\hat{\Lambda}^i(3,K)$$

$$S_{ret}(P) = \not{P}\, G_{ret}(P),\ G_{ret}(P)\ G_{adv}(P) = -\frac{\rho(P)}{\mathrm{Im}\hat{\Sigma}(P)}$$

$$\hat{\Sigma}(K) = \mathrm{Tr}(\not{K}\,\Sigma_{ret}(K))$$

$$\hat{\Lambda}^i(3,K) = \mathrm{Tr}(\not{K}\,\Lambda_{rar}(K))$$

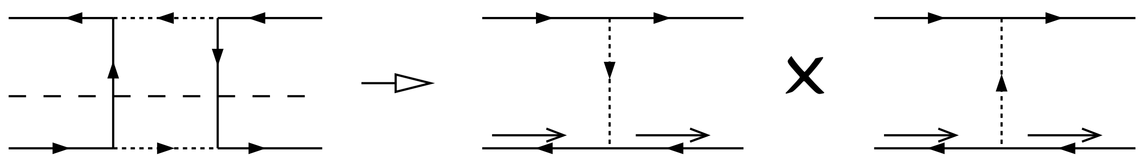
$$\hat{M}(P,K) = \mathrm{Tr}(\not{P}\,[-i\mathbf{M}(P,K)]\not{K}\,)$$

kernel $\mathrm{Re}\,[\,\hat{M}^{(j)}(P,K)\,] \rightarrow |ME|^2$

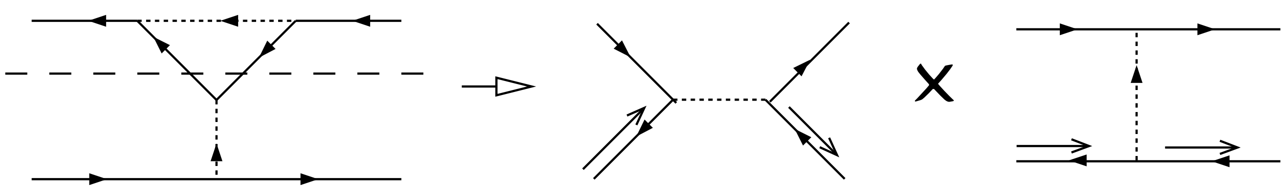
$\Phi(2-\text{loop}) \rightarrow$ the square of the s-channel
complete to leading log order
G. Aarts and J. Martinez-Resco, JHEP 03, 074 (2005).

$\Phi(3-\text{loop}) \rightarrow t$- and u-channels
part of full lo contribution
MEC and E. Kovalchuk, Phys. Rev. D **76**, *045019 (2007).*

$$\mathrm{Re}\,\hat{M}^{(f)} \quad \rightarrow \quad |m^t_{e^+e^-\rightarrow e^+e^-}|^2$$

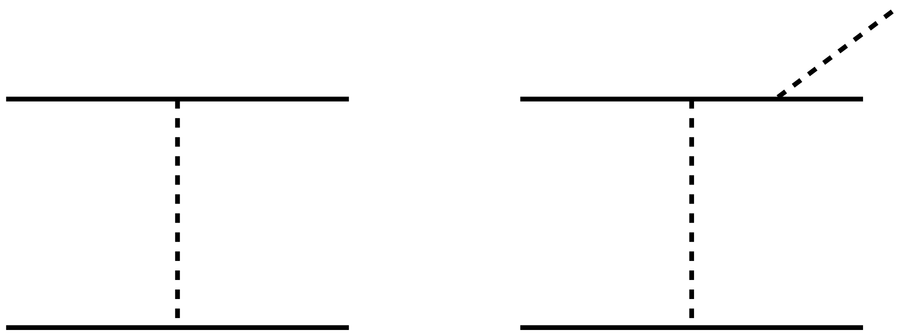


$$\mathrm{Re}\,M^{(d)} \quad \rightarrow \quad m^{t\dagger}_{e^+e^-\rightarrow e^+e^-} \cdot m^s_{e^+e^-\rightarrow e^+e^-}$$



RESUM PINCHES $\not\rightarrow$ FULL LEADING ORDER

basic idea:
compare $2 \rightarrow 2$ and $2 \rightarrow 3$



2nd is formally higher order
BUT collinear singularity $\rightarrow$ enhancement

∞ series of collinear singularities must be resummed (LPM effect)

$\Rightarrow$ need 2 coupled integral equations that resum pinching and collinear singularities

METHOD: 3PI EFFECTIVE ACTION

motivation:

need 3-loop diagram to get t- and u- channels in ME

heirarchy: $(n \to \infty)PI|_{3-\text{loop}} = 3PI|_{3-\text{loop}}$

J. Berges, Phys. Rev. D **70**, *105010 (2004).*

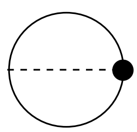
result:

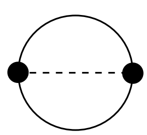
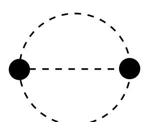
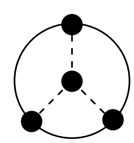
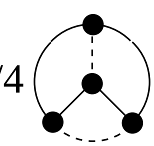
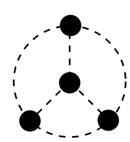
3PI $\Gamma \longrightarrow$ 2 int eqns: pinch and collinear singularities

MEC and E. Kovalchuk, Phys. Rev. D **77**, *025015 (2008).*

3PI Γ:

$$
\begin{aligned}
\Gamma[\psi,\bar{\psi},A,S,D,V,U] &= S_{cl}[\psi,\bar{\psi},A] \\
&+\frac{i}{2}\mathrm{Tr}\,\mathrm{Ln}D_{12}^{-1}+\frac{i}{2}\mathrm{Tr}\left[(D_{12}^{0})^{-1}\left(D_{21}-D_{21}^{0}\right)\right] \\
&-i\mathrm{Tr}\,\mathrm{Ln}S_{12}^{-1}-i\mathrm{Tr}\left[(S_{12}^{0})^{-1}(S_{21}-S_{21}^{0})\right] \\
&\quad+\Gamma_2^0[S,D,V,U]+\Gamma_2^{\mathrm{int}}[S,D,V,U]
\end{aligned}
$$

$\Gamma_2^0 =$ i 

$\Gamma_2^{\mathrm{int}} =$ −i/2  +i/12  +i/3  +i/4  −i/24 

RESUMMED ACTION:

7 EoM:
functional derivatives wrt $\{A, \psi, \bar{\psi}, S, D, V, U\}$

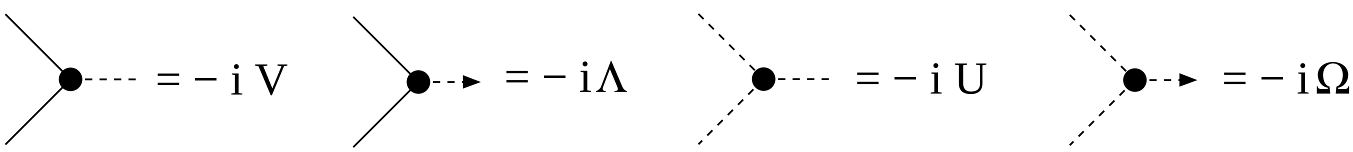
solve last 4 simultaneously for the sc solns:

$$\tilde{S}[\psi, \bar{\psi}, A], \quad \tilde{D}[\psi, \bar{\psi}, A], \quad \tilde{V}[\psi, \bar{\psi}, A], \quad \tilde{U}[\psi, \bar{\psi}, A]$$

resummed action:

$$\tilde{\Gamma}[\psi, \bar{\psi}, A] =$$
$$\Gamma[\psi, \bar{\psi}, A, \tilde{S}[\psi, \bar{\psi}, A], \tilde{D}[\psi, \bar{\psi}, A], \tilde{V}[\psi, \bar{\psi}, A], \tilde{U}[\psi, \bar{\psi}, A]]$$

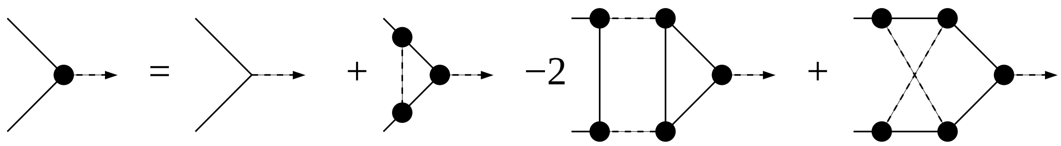
define external vertices same as before



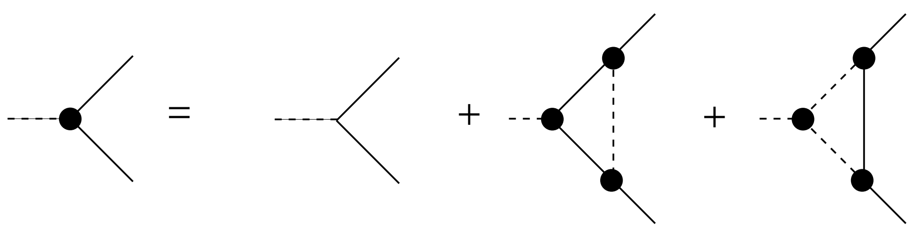
INTEGRAL EQUATIONS:

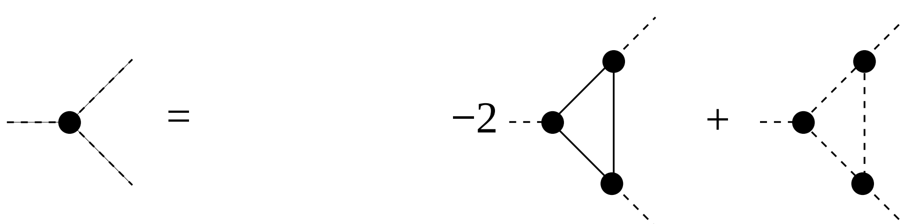
2 EoM from fcn derivs of Γ wrt S and D (SD eqns)
differentiate wrt A
$\longrightarrow$ BS eqn for Λ and Ω (many cancellations)

sub Ω eqn into Λ eqn and keep up to 2-loop order:

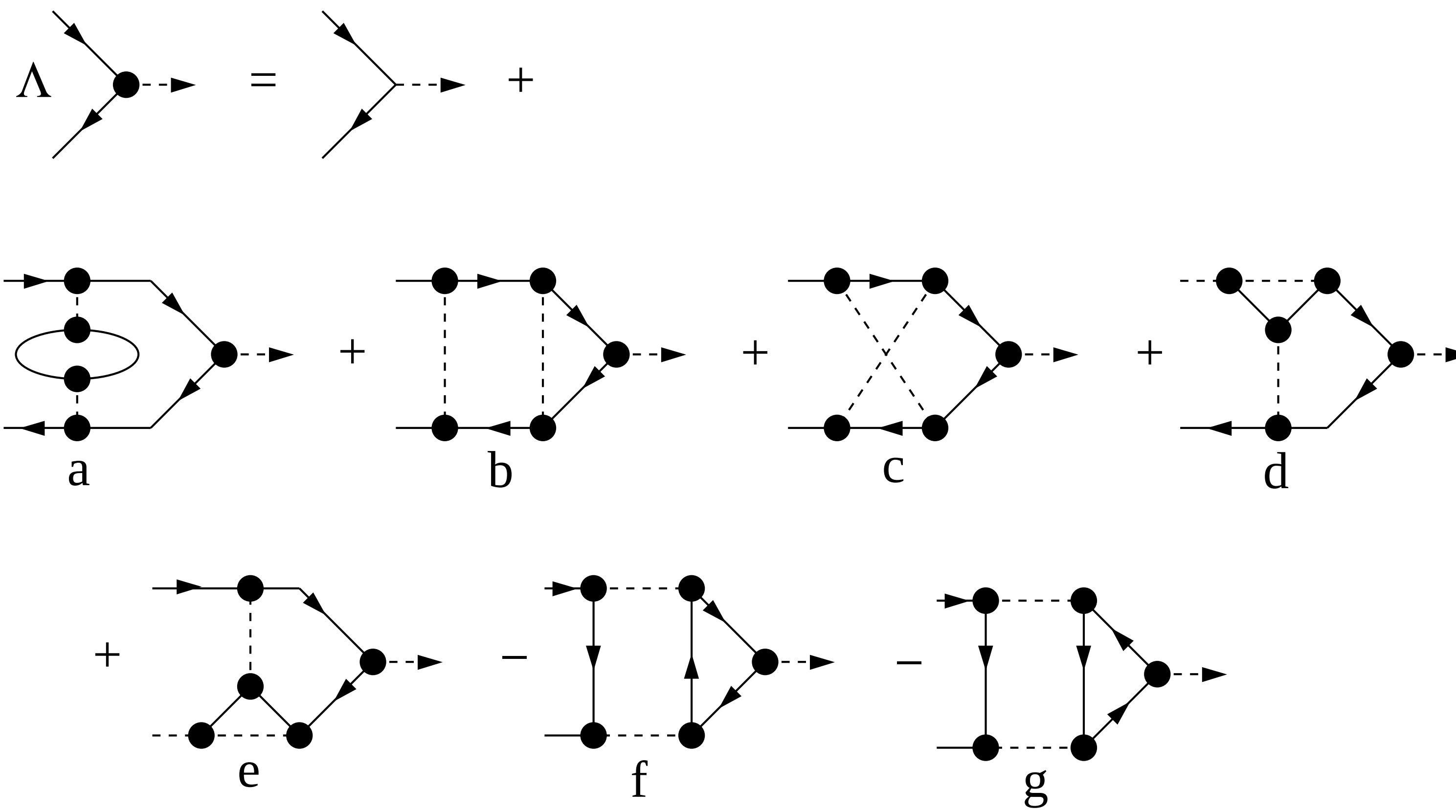


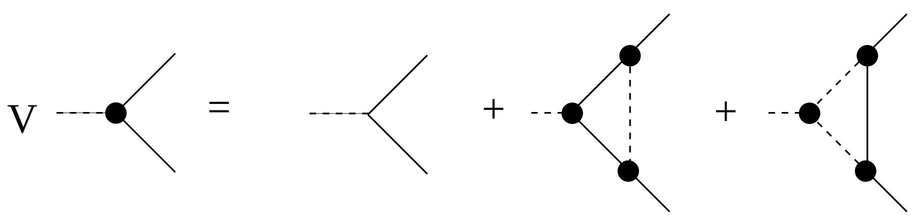
also: 2 EoM from fcn derivs of Γ wrt V and U





→ substitute again and keep up to 2-loop order:





1st is int eqn for Λ $\sim$ same as from 2PI
2nd is sc int eqn for V

same eqns found using kinetic theory (AMY)
also found using a diagramatic approach
J-S Gagnon and S. Jeon, Phys. Rev. **D75**, *025014 (2007)*

PROSPECTS

method should be generalizable to:

[1] other transport coefficients (shear viscosity)

[2] other theories (QCD)

[3] nlo (?)

KEY:

ALL LEADING ORDER TERMS APPEAR NATURALLY W/O ANY POWER COUNTING

PROBLEMS

external n-point functions satisfy wi

may get gauge dependence from sc (internal props)

`gauge invariance of the effective action:`

calculate 2PI Γ to L-loop order (g^{2L-2})

$\rightarrow$ gauge dependent terms appear at order g^{2L}

[1] from behaviour of Γ under BRS transformations

A. Arrizabalaga, J. Smit, Phys. Rev. **D66**, *065014 (2002).*

[2] from 2PI Nielsen identities

explicit gauge dependence of Γ compensates gauge dependence of the vev

MEC, G. Kunstatter, H. Zaraket, Eur.Phys.J. **C42**, *253 (2005).*

sc propagators are determined numerically from Γ
$\longrightarrow$ expect also gauge indep up to order of truncation

certainly should be okay for thermodynamic observ.

checked for 2-loop qed pressure from 2PI Γ
S. Borsanyi, U. Reinosa - arXiv:0709.2316.